NOVA SOUTHEASTERN UNIVERSITY

Abraham S. Fischler School of Education

Module 7: ANOVA

The Applied Research Center

Module 7 Overview

- Analysis of Variance
- Types of ANOVAs
  - One-way ANOVA
  - Two-way ANOVA
  - MANOVA
  - ANCOVA

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One-way ANOVA

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ANOVA

- Analysis of variance
- Used to test 3 or more means
- Used to test the null hypothesis that several means are equal
- For example:
  - $H_0: \mu_1 = \mu_2 = \mu_3$
  - $H_a: \mu_1 \neq \mu_2 \neq \mu_3$ or $H_a: \mu_1 > \mu_2 > \mu_3$

Different types of ANOVAs

- One-way ANOVA
  - one IV (more than two levels)
- Two-way ANOVA
  - two IVs
- RM ANOVA
  - repeated measures on one or more factors
- MANOVA
  - multiple DVs

One-way ANOVA

- Example:
  - A stats teacher wants to know if there is a significant difference in grades for assignments 1, 2, and 3 in her stats class.
  - **NOTE**: the assignments could not be matched, therefore, a RM ANOVA was not appropriate.

One-way ANOVA (cont'd)

- Step 1: Write the H_o and H_a hypotheses
  - H_o: The means for Assignment 1, Assignment 2, and Assignment 3 are equal.
    - $H_0: \mu_1 = \mu_2 = \mu_3$
  - H_a: The means for Assignment 1, Assignment 2, and Assignment 3 are not equal.
    - $H_a: \mu_1 \neq \mu_2 \neq \mu_3$

One-way ANOVA (cont'd)

- Step 2: Input each student's grade into SPSS
- Run the Analysis:
  - Analyze → Compare Means → One-way ANOVA
  - Dependent List = Grade
  - Factor = Assign#
  - Click Options and select Descriptive, click continue
  - Click OK

One-way ANOVA (cont' d)

Descriptives

Grade

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Assignment 1	15	21.0333	1.54072	.39781	20.1801	21.8866	18.50	23.50
Assignment 2	13	22.5385	1.19829	.33235	21.8143	23.2626	20.50	24.50
Assignment 3	13	23.4615	1.68895	.46843	22.4409	24.4822	20.00	25.00
Total	41	22.2805	1.78202	.27831	21.7180	22.8430	18.50	25.00

ANOVA

Grade

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	42.330	2	21.165	9.496	.000
Within Groups	84.695	38	2.229		
Total	127.024	40			

One-way ANOVA (cont'd)

- Step 4: Make a decision regarding the null
  - Assignment 1: $M = 21.03$, $SD = 1.54$
  - Assignment 2: $M = 22.54$, $SD = 1.20$
  - Assignment 3: $M = 23.46$, $SD = 1.54$
  - $F(2, 38) = 9.50$
  - $p < .001$

- df = (df between, df within)
  - df b/n = k-1 = 3-1 = 2
  - df within = $[(n_1 - 1) + [(n_2 - 1) + [(n_3 - 1)] = 14 + 12 + 12 = 38$
- What is the decision regarding the null?

One-way ANOVA (cont' d)

- Using the level of significance = .05, do we reject or fail to reject the null?
  - If $p < .05$, we reject the null
  - if $p > .05$, we fail to reject the null
- According to SPSS, $p < .001$
- $.001 < .05$, therefore, we reject the null!

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One-way ANOVA (cont'd)

- We reject the null that said the means for Assignment 1, Assignment 2, and Assignment 3 are equal.
- Therefore, the means are not equal.
- How do we know which means are different?

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Post hoc comparisons

- In addition to determining that differences exist among the means, you can also look at which means differ after the fact.
- Most common post hoc comparisons:
  - Fisher's LSD (Least sig diff)
  - Tukey's HSD (Honestly sig diff)

One-way ANOVA (cont'd)

- Step 5: Post hoc analyses
- Using Fisher's LSD post hoc comparison:
  - Analyze → Compare Means → One-way ANOVA
  - Dependent List = Grade
  - Factor = Assign#
  - Click Options, select Descriptive, click continue
  - Click Post Hoc, select LSD, click continue
  - Click OK

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One-way ANOVA (cont' d)

Multiple Comparisons

Dependent Variable: Grade
LSD

(I) Assign#	(J) Assign#	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Assignment 1	Assignment 2	-1.50513*	.56572	.011	-2.6504	-.3599
	Assignment 3	-2.42821*	.56572	.000	-3.5734	-1.2830
Assignment 2	Assignment 1	1.50513*	.56572	.011	.3599	2.6504
	Assignment 3	-.92308	.58557	.123	-2.1085	.2624
Assignment 3	Assignment 1	2.42821*	.56572	.000	1.2830	3.5734
	Assignment 2	.92308	.58557	.123	-.2624	2.1085

*. The mean difference is significant at the .05 level.

One-way ANOVA (cont'd)

- Which effects are significant?
- Remember, the nulls here say the 2 means are equal, therefore there are 3 nulls
  - $H_o: A1 = A2; H_a: A1 \neq A2$
  - $H_o: A1 = A3; H_a: A1 \neq A3$
  - $H_o: A2 = A3; H_a: A2 \neq A3$
- A1 – A2, $p = .01$
- A1 – A3, $p < .001$
- A2 – A3, $p = .123$

One-way ANOVA (cont' d)

- Using the level of significance = .05, do we reject or fail to reject the null?
  - If $p < .05$, we reject the null
  - if $p > .05$, we fail to reject the null

- A1 – A2, $p = .01 < .05$; reject null
- A1 – A3, $p < .001 < .05$, reject null
- A2 – A3, $p = .123 > .05$, fail to reject null

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One-way ANOVA (cont'd)

- Step 6: Write up your results.

- The null hypothesis stated that the means for Assignment 1, Assignment 2, and Assignment 3 are equal. A One-way ANOVA revealed a significant difference among the means for the 3 assignments, $F\ (2, 38) = 9.50, p < .001, \eta^2 = .33$. Students' grades on A1 (M = 21.03, SD = 1.54) were significantly lower than A2 (M = 22.54, SD = 1.20; p = .01), and A3 (M = 23.46, SD = 1.54; p < .001). There was no significant difference in students' grades between A2 and A3 (p = .12).

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Partial eta squared (η^2)

- Measure of effect size
- Interpretation: The percentage of variance in each of the effects (or interaction) and its associated error that is accounted for by that effect.
- Used as a comparison to other studies (rather than typical cut-off values as in Cohen's *d*).

Partial eta squared (η^2)

- To obtain:
  - Analyze → General Linear Model → Univariate
  - Dependent Variable = Grade
  - Fixed Factor = Assign#
  - Click Options, select
    - Descriptive statistics
    - Estimates of effect size
    - Click Continue
  - Click OK

Univariate Analysis of Variance

Between-Subjects Factors

		Value Label	N
Assign#	1.00	Assignment 1	15
	2.00	Assignment 2	13
	3.00	Assignment 3	13

Descriptive Statistics

Dependent Variable: Grade

Assign#	Mean	Std. Deviation	N
Assignment 1	21.0333	1.54072	15
Assignment 2	22.5385	1.19829	13
Assignment 3	23.4615	1.68895	13
Total	22.2805	1.78202	41

Univariate Analysis of Variance

Tests of Between-Subjects Effects

Dependent Variable: Grade

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	42.330[a]	2	21.165	9.496	.000	.333
Intercept	20377.354	1	20377.354	9142.696	.000	.996
Assign#	42.330	2	21.165	9.496	.000	.333
Error	84.695	38	2.229			
Total	20480.250	41				
Corrected Total	127.024	40				

a. R Squared = .333 (Adjusted R Squared = .298)

- This procedure produces the exact same results!!

Two-way ANOVA

- 2 IVs
- Example:
  - A stats teacher wants to determine whether students in Class A differ from students in Class B with regards to their grades on Assignments 1 and 2.
  - If can match student grades on A1 and A2, then should be ran as a RM ANOVA.

Two-way ANOVA

- Step1: Write the H_o and H_a hypotheses
- Ho: There is no difference between class and assignment number on students' grades.
- Ho: There is a difference between class and assignment number on students' grades.

Two-way ANOVA (cont’d)

- Step 2: Input each student’s grade into SPSS and
- Run the Analysis:
  - Analyze → GLM → Univariate
    - Dependent Variable = grade
    - Fixed Factors = class, assignment # (these are your IVs)
  - Click Options and select
    - Descriptives
    - Estimates of effect size
    - Homogeneity Tests
  - Click Continue

Two-way ANOVA (cont’d)

- Click Plots
  - Move Class to Horizontal Axis
  - Move Assign # to Separate Lines
  - Then select “Model” or “ADD” Button
  - Click Continue
- Click Continue; Click OK

- Do we need to run post hoc tests??

Two-way ANOVA (cont’d)

Between-Subjects Factors

		N
class	1.00	28
	2.00	25
assign#	1.00	25
	2.00	28

Descriptive Statistics

Dependent Variable:grade

class	assign#	Mean	Std. Deviation	N
1.00	1.00	22.5385	1.19829	13
	2.00	21.0333	1.54072	15
	Total	21.7321	1.56632	28
2.00	1.00	23.3333	.86164	12
	2.00	21.9038	1.93525	13
	Total	22.5900	1.65655	25
Total	1.00	22.9200	1.10567	25
	2.00	21.4375	1.75808	28
	Total	22.1368	1.65146	53

Two-way ANOVA (cont’d)

Levene's Test of Equality of Error Variances[a]

Dependent Variable:grade

F	df1	df2	Sig.
6.768	3	49	.001

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + class + assign# + class * assign#

Two-way ANOVA (cont' d)

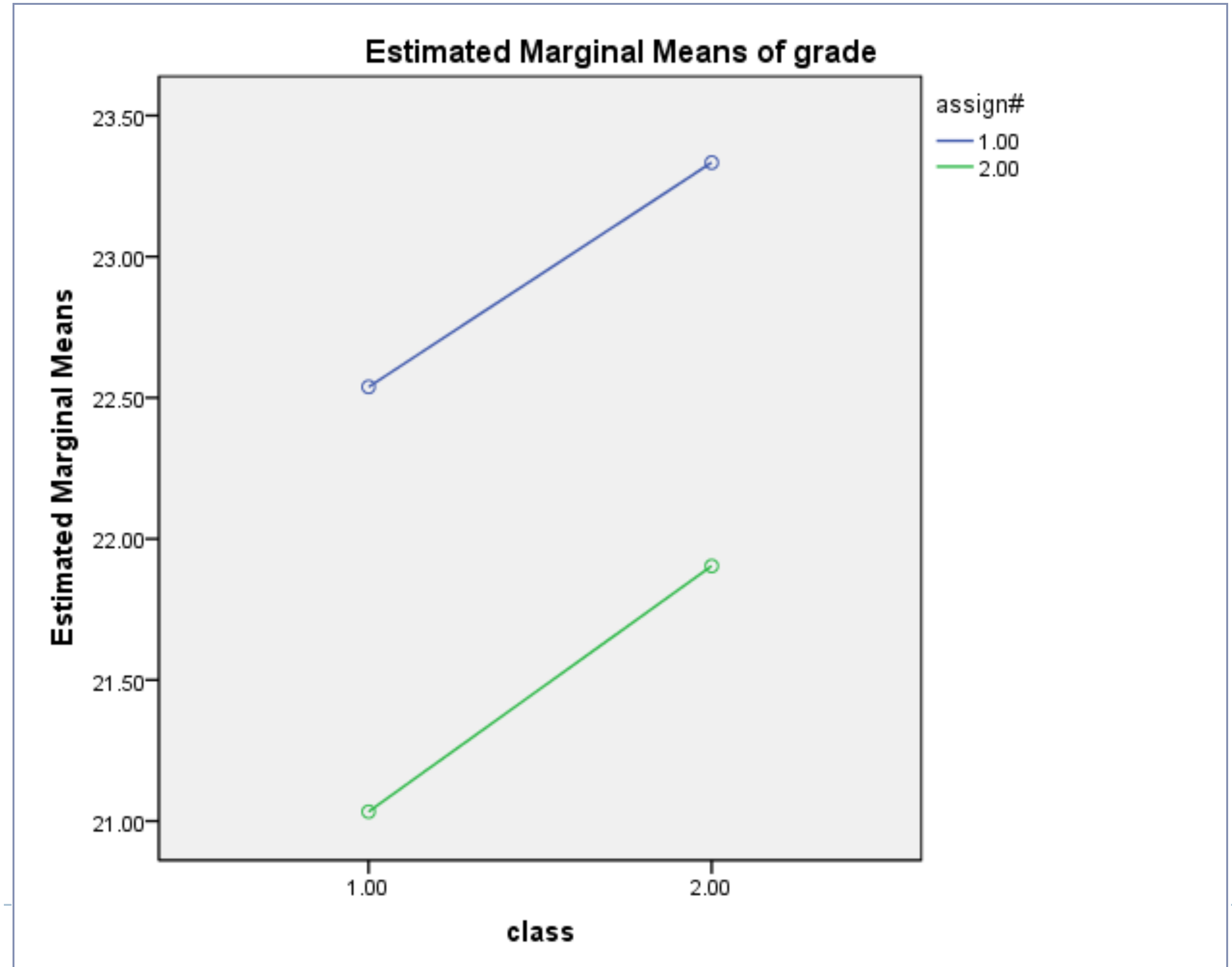
Tests of Between-Subjects Effects

Dependent Variable:grade

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	38.248[a]	3	12.749	6.032	.001	.270
Intercept	25957.327	1	25957.327	12280.305	.000	.996
class	9.128	1	9.128	4.318	.043	.081
assign#	28.343	1	28.343	13.409	.001	.215
class * assign#	.019	1	.019	.009	.925	.000
Error	103.573	49	2.114			
Total	26113.813	53				
Corrected Total	141.821	52				

a. R Squared = .270 (Adjusted R Squared = .225)

Two-way ANOVA (cont’d)



Two-way ANOVA (cont'd)

- Step 3: Make a decision regarding the null.
- Do we reject or fail to reject the null?

Two-way ANOVA (cont' d)

- Step 4: Write up your results.
- The null hypothesis stated that there is no difference between class and assignment number on students' grades. A Two-way ANOVA revealed a significant difference between classes (M = 21.73, SD = 1.57; M = 22.59, SD = 1.66; for Class 1 and 2, respectively) on students' grades, $F(1, 49) = 4.32$, $p = .04$, $\eta^2 = .08$, and between assignment number (M = 22.92, SD = 1.11; M = 21.44, SD = 1.76; for Assignment 1 and 2, respectively) and students' grades, $F(1, 49) = 13.41$, $p = .001$, $\eta^2 = .22$; however, the grades by class interaction effect was not significant, $F(1, 49) = .01$, $p = .93$, $\eta^2 = .00$.

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MANOVA

- 2 or more DVs
- Example:
  - A stats teacher wants to determine whether students in Class A differ from students in Class B on Assignment 1 and their anxiety towards statistics (based on a survey given at the beginning of the semester).

MANOVA

- To run, Analyze → GLM → Mulitvariate
  - Dependent Variables = grade, anxiety score (2 DVs)
  - Fixed Factors = class, assignment # (these are your IVs)

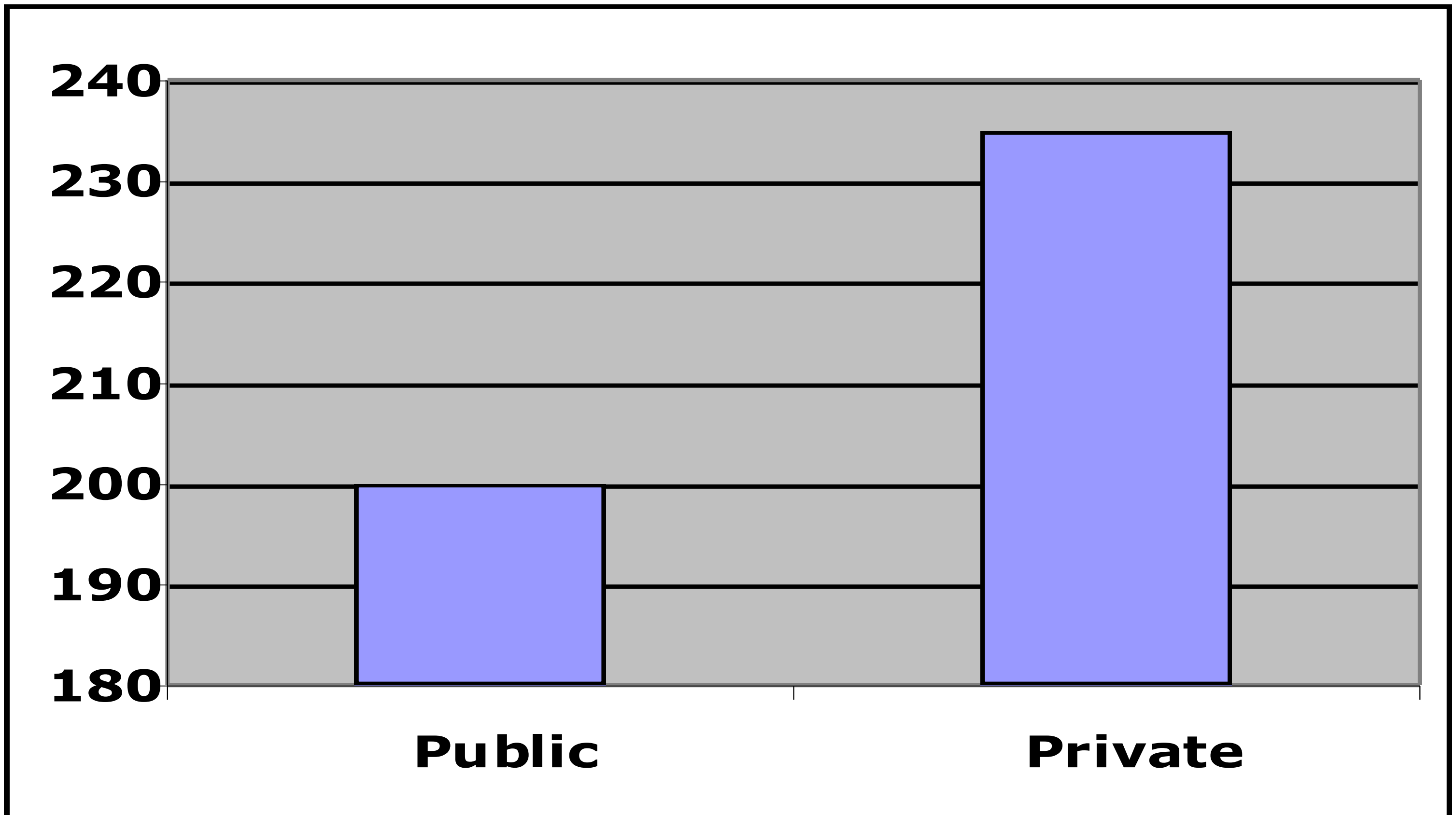
ANCOVA

- In quasi-experimental designs random assignment of subjects is not possible (e.g., using a non-equivalent control group)
- What's the biggest problem with these types of designs?
- We can control this through our data analysis by including a covariate

ANCOVA Example

- Often times we want to evaluate the effectiveness of a program that is already in place, and we are not able to construct a treatment and a control group.
- For example, suppose we wanted to evaluate the effectiveness of public schools vs. private schools on academic achievement. We looked at the average NAEP math scores for 4th grade students in public and private schools and found the following:

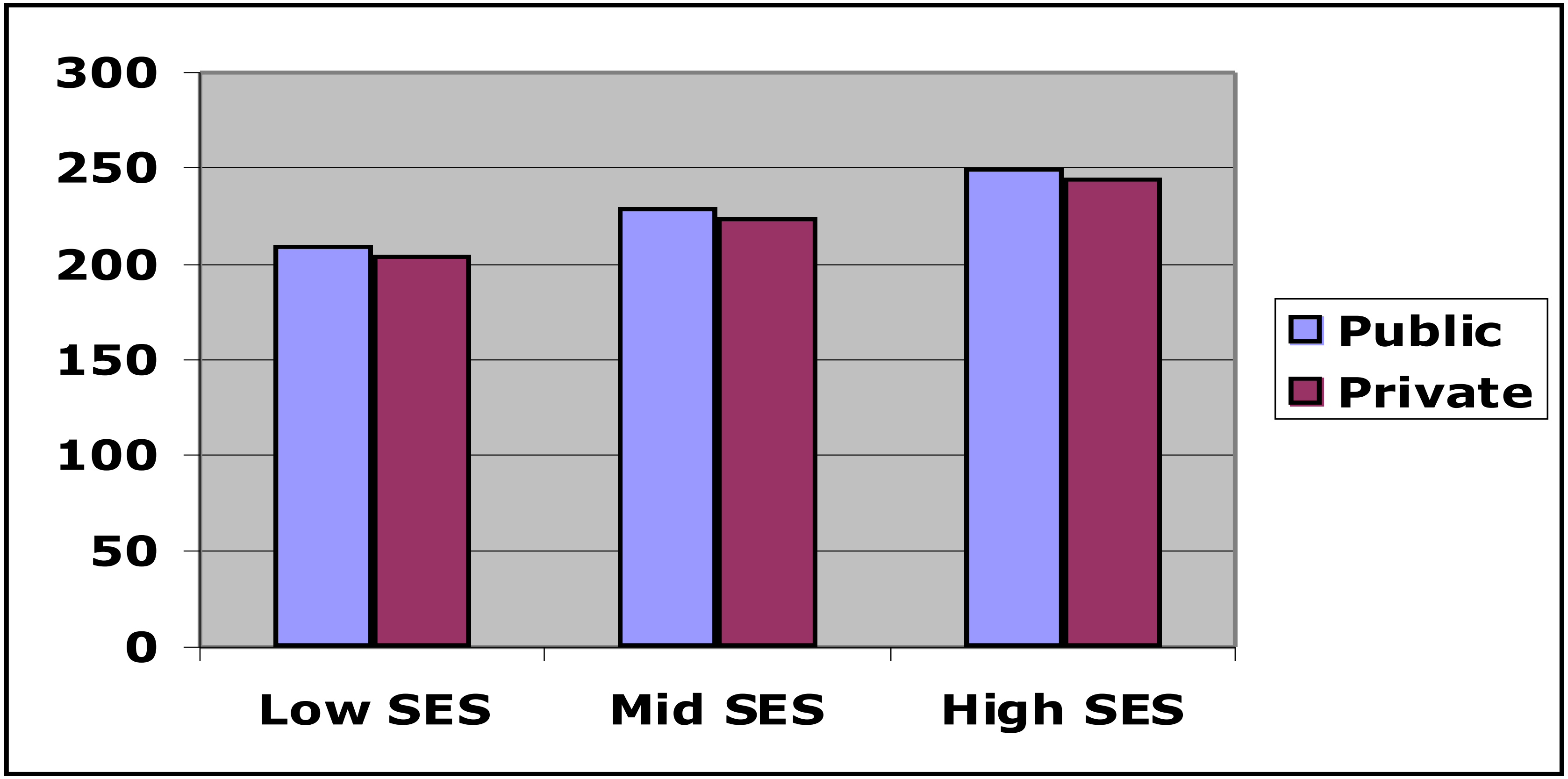
ANCOVA Example (cont’d)



ANCOVA Example (cont'd)

- What happens when we control for an extraneous variable such as SES (i.e., use SES as a covariate).

ANCOVA Example (cont'd)



ANCOVA Example (cont’d)

- When we compare public and private students of the same SES, we find there is little difference in their achievement. But because there are more high SES students in private schools, the overall comparison is misleading.

ANCOVA Example (cont’d)

- ANCOVAs are run similarly to ANOVAs, you simply add the variable as a covriate.
- To run, Analyze → GLM → Univariate
  - Covariate = SES
- Interpreted the same way as the ANOVA output

Module 7 Summary

- Analysis of Variance
- Types of ANOVAs
  - One-way ANOVA
  - Two-way ANOVA
  - MANOVA
  - ANCOVA

Review Activity

- Please complete the review activity at the end of the module.
- All modules build on one another. Therefore, in order to move onto the next module you must successfully complete the review activity before moving on to next module.
- You can complete the review activity and module as many times as you like.

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Upcoming Modules

- Module 1: Introduction to Statistics
- Module 2: Introduction to SPSS
- Module 3: Descriptive Statistics
- Module 4: Inferential Statistics
- Module 5: Correlation
- Module 6: *t*-*Tests*
- Module 7: ANOVAs
- Module 8: Linear Regression
- Module 9: Nonparametric Procedures